2011年度数学Ⅰ・A 問題

第1問(配点20点)

［1］ $a = 3 + 2\sqrt{2}$, $b = 2 + \sqrt{3}$ とすると

$$\frac{1}{a} = \boxed{ア} - \boxed{イ}\sqrt{\boxed{ウ}}$$

$$\frac{1}{b} = \boxed{エ} - \sqrt{\boxed{オ}}$$

$$\frac{a}{b} - \frac{b}{a} = \boxed{カ}\sqrt{\boxed{キ}} - \boxed{ク}\sqrt{\boxed{ケ}}$$

［Tメソ］

$$\frac{1}{a} = \frac{1}{3 + 2\sqrt{2}}$$

$$= \frac{1 \times (3 - 2\sqrt{2})}{(3 + 2\sqrt{2})(3 - 2\sqrt{2})}$$

$$= \frac{3 - 2\sqrt{2}}{3^2 - (2\sqrt{2})^2}$$

$$= \frac{3 - 2\sqrt{2}}{3 \times 3 - (2\sqrt{2} \times 2\sqrt{2})}$$

$$= \frac{3 - 2\sqrt{2}}{9 - (2 \times \sqrt{2} \times 2 \times \sqrt{2})}$$

$$= \frac{3 - 2\sqrt{2}}{9 - (2 \times 2 \times \sqrt{2} \times \sqrt{2})}$$

$$= \frac{3 - 2\sqrt{2}}{9 - (4 \times 2)}$$

$$= \frac{3 - 2\sqrt{2}}{9 - (8)}$$

$$= \frac{3 - 2\sqrt{2}}{9 - 8}$$

$$= \frac{3 - 2\sqrt{2}}{1}$$

$$= 3 - 2\sqrt{2}$$

$$\frac{1}{b} = \frac{1}{2 + \sqrt{3}}$$

$$= \frac{1 \times (2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})}$$

$$= \frac{2 - \sqrt{3}}{2^2 - (\sqrt{3})^2}$$

$$= \frac{2 - \sqrt{3}}{2 \times 2 - \sqrt{3} \times \sqrt{3}}$$

$$= \frac{2 - \sqrt{3}}{4 - 3}$$

$$= \frac{2 - \sqrt{3}}{1}$$

$$= 2 - \sqrt{3}$$

$$\frac{a}{b} - \frac{b}{a} = \frac{a \times 1}{1 \times b} - \frac{b \times 1}{1 \times a}$$

$$= \frac{a}{1} \times \frac{1}{b} - \frac{b}{1} \times \frac{1}{a}$$

$$= a \times \frac{1}{b} - b \times \frac{1}{a}$$

$\frac{1}{b} = 2 - \sqrt{3}$, $\frac{1}{a} = 3 - 2\sqrt{2}$

$$= (3 + 2\sqrt{2}) \times (2 - \sqrt{3}) - (2 + \sqrt{3}) \times (3 - 2\sqrt{2})$$

$$= (6 - 3\sqrt{3} + 4\sqrt{2} - 2\sqrt{6}) - (6 - 4\sqrt{2} + 3\sqrt{3} - 2\sqrt{6})$$

$$= 6 - 3\sqrt{3} + 4\sqrt{2} - 2\sqrt{6} - 6 + 4\sqrt{2} - 3\sqrt{3} + 2\sqrt{6}$$

$$= 6 - 6 + 4\sqrt{2} + 4\sqrt{2} - 3\sqrt{3} - 3\sqrt{3} - 2\sqrt{6} + 2\sqrt{6}$$

$$= 8\sqrt{2} - 6\sqrt{3} \quad \cdots\cdots ①$$

このとき，不等式

$$|2abx - a^2| < b^2$$

を満たすxの値の範囲は

$$\boxed{コ}\sqrt{\boxed{サ}} - \boxed{シ}\sqrt{\boxed{ス}} < x < \boxed{セ} - \boxed{ソ}\sqrt{\boxed{タ}}$$

となる。

［Tメソ］

$|2abx - a^2| < b^2$より　　$2abx - a^2 > 0$ であれば　$2abx - a^2 < b^2$

$2abx - a^2 < 0$ であれば　$-(2abx - a^2) < b^2$

$2abx - a^2 > -b^2$

両辺に負をかけると不等号の向きが逆になる。

例　$|-3| < 2^2$　であれば　$-(-3) < 2^2$

$-3 > -2^2$

$-3 > -4$

$$-b^2 < 2abx - a^2 < b^2$$

$$-b^2 + a^2 < 2abx - a^2 + a^2 < b^2 + a^2$$

$$a^2 - b^2 < 2abx < a^2 + b^2$$

$a = 3 + 2\sqrt{2}$, $b = 2 + \sqrt{3}$

$ab > 0$ なので abで両辺を割っても不等号の向きは変わらない

$$\frac{a^2 - b^2}{2ab} < \frac{2abx}{2ab} < \frac{a^2 + b^2}{2ab}$$

$$\frac{a^2 - b^2}{2ab} < x < \frac{a^2 + b^2}{2ab}$$

ここで

$$\frac{a^2 - b^2}{2ab} = \frac{a^2}{2ab} - \frac{b^2}{2ab}$$

$$= \frac{a}{2b} - \frac{b}{2a}$$

$$= \frac{1}{2}\left(\frac{a}{b} - \frac{b}{a} \right)$$ ①を代入

$$= \frac{1}{2}(8\sqrt{2} - 6\sqrt{3})$$ $a = 3 + 2\sqrt{2}$, $b = 2 + \sqrt{3}$

$$= 4\sqrt{2} - 3\sqrt{3}$$

$$\frac{a^2 + b^2}{2ab} = \frac{a^2}{2ab} + \frac{b^2}{2ab}$$

$$= \frac{a}{2b} + \frac{b}{2a}$$

$$= \frac{1}{2}\left(\frac{a}{b} + \frac{b}{a} \right)$$

$a = 3 + 2\sqrt{2}$, $b = 2 + \sqrt{3}$

$$= \frac{1}{2}\left(a \times \frac{1}{b} + b \times \frac{1}{a} \right)$$ $\frac{1}{a} = 3 - 2\sqrt{2}$ $\frac{1}{b} = 2 - \sqrt{3}$

$$= \frac{1}{2}\left\{ (3 + 2\sqrt{2})(2 - \sqrt{3}) + (2 + \sqrt{3})(3 - 2\sqrt{2}) \right\}$$

$$= \frac{1}{2}\left\{ (6 - 3\sqrt{3} + 4\sqrt{2} - 2\sqrt{6}) + (6 - 4\sqrt{2} + 3\sqrt{3} - 2\sqrt{6} \right\}$$

$$= \frac{1}{2}(6 - 3\sqrt{3} + 4\sqrt{2} - 2\sqrt{6} + 6 - 4\sqrt{2} + 3\sqrt{3} - 2\sqrt{6})$$

$$= \frac{1}{2}(6 + 6 + 4\sqrt{2} - 4\sqrt{2} - 3\sqrt{3} + 3\sqrt{3} - 2\sqrt{6} - 2\sqrt{6})$$

$$= \frac{1}{2}(12 - 4\sqrt{6})$$

$$= 6 - 2\sqrt{6}$$

$$\frac{a^2 - b^2}{2ab} = 4\sqrt{2} - 3\sqrt{3}$$

$$\frac{a^2 + b^2}{2ab} = 6 - 2\sqrt{6}$$

$$\frac{a^2-b^2}{2ab} < x < \frac{a^2+b^2}{2ab}$$ に代入

$$4\sqrt{2} - 3\sqrt{3} < x < 6 - 2\sqrt{6}$$